

SUBJECT:

**PHYSICS**

CLASS:

**SENIOR SECONDARY SCHOOL 2**

TERM:

**FIRST**

## **SCHEME OF WORK**

<b>WEEK</b>	<b>TOPIC</b>
1.	Position, distance and displacement.
2.	Scalar and vector Quantities-Concept of scalar and vector quantities, vector representation etc.
3.	Derivation of equation of linear motion, Motion under gravity, calculation using these equations.
4.	Projectiles and its application.
5.	Newton Laws of Motion - Conservation of Linear momentum and collision energy.
6.	Equilibrium if Forces - principle of moment, conditions for equilibrium of a Rigid Bodies etc.
7.	Equilibrium if Forces - centre of gravity and stability, couple
<b>MID-TERM PROJECT</b>	
8.	Simple Harmonic Motion- definition, speed, amplitude, displacement, acceleration, etc.
9.	Simple Harmonic Motion - Energy of simple harmonic motion and forced vibration, Resonance.
10.	Machines - Types and Examples
11.	Machines - Calculation.
12.	Revision
13.	Examination

## **WEEK 1**

### **POSITION, DISTANCE AND DISPLACEMENT**

#### **CONTENTS**

- Position
- Distance
- Displacement

#### **POSITION**

The position of an object in space or on a plane is the point at which the object can be located with reference to a given point (the origin).

#### **DISTANCE**

This is a measure of the separation between two points. It has magnitude but no direction. Hence, it is a scalar quantity

#### **DETERMINATION OF DISTANCE BETWEEN TWO POINTS**

If two points A and B located in a plane are defined by two ordered pair of values  $(X_1, Y_1)$  and  $(X_2, Y_2)$  or assumed to be in space where they are defined by  $(X_1, Y_1, Z_1)$  and  $(X_2, Y_2, Z_2)$  the distance between them can be determined by applying this relation.

$$S = \sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2}$$

OR

$$S = \sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2 + (Z_2 - Z_1)^2}$$

## **DISPLACEMENT**

Displacement is the distance covered in a specified direction. It is a vector quantity, which has the same unit as distance.

## **CLASSWORK**

1. What is displacement and why is it regarded as vector quantity?
2. Highlight three differences between distance and displacement

## **ASSIGNMENT**

### **SECTION A**

1. Which of the following is odd? (a) 25km (b) 25km North (c) 25km to the left (d) 25km upwards
2. The following are vector quantities EXCEPT (a) distance (b) displacement (c) force (d) weight
3. Determine the distance between S (3,4,-5) and T (2,1,0) (a) 5.8 (b) 5.9 (c) 6.0 (d) 6.2
4. Distance is can be measure with the following except (a) metre rule (b) speedometer (c) tape rule (d) ruler
5. Which is the correct SI unit of distance? (a) millimeters (b) metres (c) kilometres (d) centimetres

### **SECTION B**

1. Sketch clearly using scale indicators, the position of a point P (4,-5,6) with reference to a point Q (0,0,0). Determine the distance between P and Q
2. What is position?

## **WEEK 2**

### **SCALAR AND VECTOR QUANTITIES**

#### **CONTENTS**

- Concept of scalar and vector quantities
- Vector representation, addition of vectors
- Resolution of vectors and resultant

#### **CONCEPT OF SCALAR AND VECTOR QUANTITIES**

Physical quantities are divided into scalar and vector quantities.

A scalar is one which has only magnitude (size) e.g. distance, speed, temperature, volume, work, energy, power, mass etc.

A vector quantity has both magnitude and direction e.g. force, weight, magnetic flux, electric fields, gravitational fields etc.

#### **VECTOR REPRESENTATION**

A vector quantity can be graphically represented by a line drawn so that the length of the line denotes the magnitude of the quantity. The direction of the vector is shown by the arrow head.

#### **ADDITION AND SUBTRACTION OF VECTORS**

Two or more vectors acting on a body in a specified direction can be combined to produce a single vector having the same effect. The single vector is called the resultant.

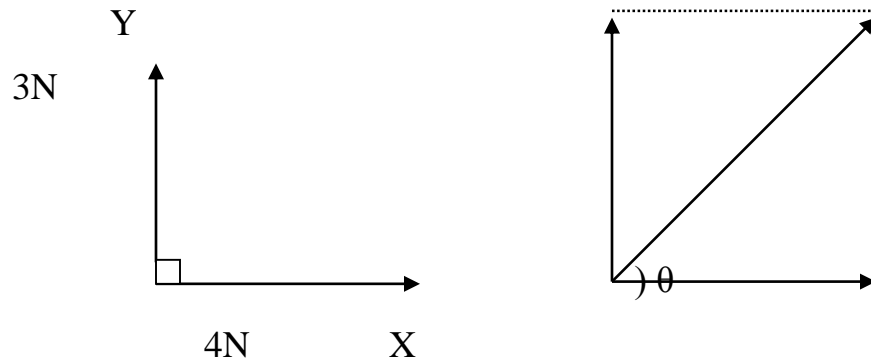


For example:

(a) Two forces Y and X with magnitude of 3N and 4N respectively acting along the same direction will produce a resultant of 7N (algebraic sum of the two vectors).

(b) If Y and X act in opposite direction, the resultant will be 1N.

(c) If the two vectors are inclined at  $90^\circ$  to each other, Pythagoras theorem is used.



$$R^2 = X^2 + Y^2$$

$$R^2 = 4^2 + 3^2$$

$$R^2 = 16 + 9$$

$$R^2 = 25$$

$$R = \sqrt{25}$$

$$R = 5\text{N}$$

$$\tan \theta = Y/X$$

$$\theta = \tan^{-1}(Y/X)$$

$$\theta = \tan^{-1}(3/4)$$

$$\theta = \tan^{-1}(0.75)$$

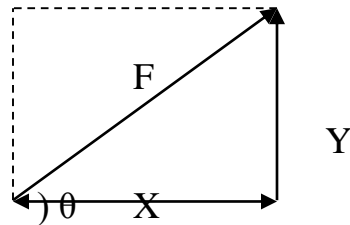
$$\theta = 36.9^\circ$$

(d) If the two vectors are inclined at an angle less than  $90^\circ$  or more than  $90^\circ$ , the resultant is obtained by using Parallelogram law of vector addition.

*Parallelogram law of vector addition states that if two vectors are represented in magnitude and direction by adjacent sides of a parallelogram, the resultant is represented in magnitude and direction by the diagonal of the parallelogram drawn from the common point*

## RESOLUTION OF VECTORS

A single vector can be resolved into two vectors called components. A vector  $F$  represented as the diagonal of the parallelogram can be resolved into its component later taken as the adjacent sides of the parallelogram.



$$\sin \theta = y / F$$

$$y = f \sin \theta \text{ (vertical component)}$$

$$\cos \theta = x / F$$

$$x = F \cos \theta \text{ (horizontal component)}$$

The direction of  $F$  is given by

$$\tan \theta = y/x$$

$$\theta = \tan^{-1} (y/x)$$

## THE RESULTANT OF MORE THAN TWO VECTORS

To find the resultant of more than two vectors, we resolve each vector in two perpendicular directions and add all the horizontal components  $X$ , and all the vertical components,  $Y$ .

For example, consider four forces acting on a body as shown below

Figure 1:

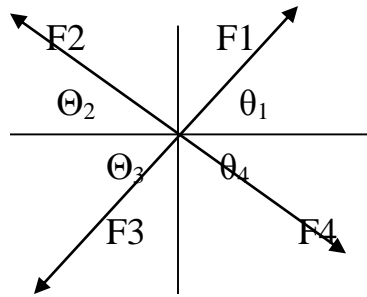
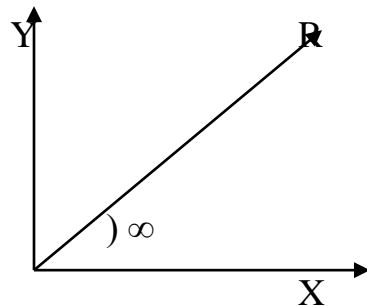


Figure 2:



Add all the resolved horizontal components

Figure 1:

$$X = F_1 \cos \theta_1 + (-F_2 \cos \theta_2) + (-F_3 \cos \theta_3) + F_4 \cos \theta_4$$

$$Y = F_1 \sin \theta_1 + F_2 \sin \theta_2 + (-F_3 \sin \theta_3) + (-F_4 \sin \theta_4)$$

Figure 2:

$$R^2 = X^2 + Y^2$$

$$R = \sqrt{X^2 + Y^2}$$

And the direction  $\infty$  is given by

$$\tan \infty = y/x$$

## CLASSWORK

1. Define vector
2. What is the difference between scalar and vector
3. Find the vertical and horizontal components of 500N force when it is inclined at (i)  $60^\circ$  (ii)  $90^\circ$  (iii)  $150^\circ$  to the ground level



## ASSIGNMENT

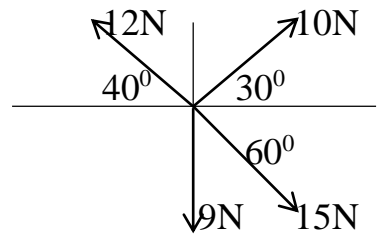
### SECTION A

1. Two forces, whose resultant is 100N, are perpendicular to each other. If one of them makes an angle of  $60^\circ$  with the resultant, calculate its magnitude: (a) 200.0N (b) 173.2N (c) 86.6 (d) 115.5
2. A boy pulling a load of 150N with a string inclined at an angle of  $30^\circ$  to the horizontal. If the tension in the string is 105N, the force tending to lift the load off the ground is: (a) 52.5N (b) 202.5N (c) 75N (d) 255N
3. A lorry travels 10km northwards, 4km eastwards, 6km southwards and 4km westwards to arrive at a point T. What is the total displacement? (a) 6km east (b) 4km north (c) 6km north (d) 4km east
4. The resultant of two forces acting on an object is maximum when the angle between them is (a)  $180^\circ$  (b)  $90^\circ$  (c)  $45^\circ$  (d)  $0^\circ$
5. A boy pulls his toy on a smooth horizontal surface with a rope inclined at  $60^\circ$  to the horizontal. If the effective force pulling the toy along the tension in rope (a) 2.5 N (b) 4.33N (c) 5.0 N (d) 8.66N (e) 10.0N

### SECTION B

1. A body of weight W newton rests on a smooth plane inclined at an angle  $\theta$  to the horizontal. What is the resolved part of the weight in newton along the plane?
2. A lawn-mower is pushed with a force 50N. If the angle between the handle of the mower and the ground is  $30^\circ$ , (a) calculate the magnitude of the force that is pressing the lawn-mower directly into the ground (b) calculate the effective force that moves the mower forward (c) why does the lawn mower move forward and not downward into the ground?

3. Calculate the resultant force in the diagram



## WEEK 3

### DERIVATION OF EQUATIONS OF LINEAR MOTION

#### CONTENTS

- Basic definitions
- Derivation of equations of linear motion
- Motion under gravity

#### BASIC DEFINITIONS

1. *Displacement*: This is the distance traveled in a specified direction. It is a vector quantity. Its unit is metres
2. *Distance*: This is the space or separation between two points. It is a scalar quantity. Its unit is metres
3. *Speed*: this is the rate of change of distance with time. It is a scalar quantity. Its unit is metre per seconds (m/s)

$$\text{Speed} = \frac{\text{distance}}{\text{Time}}$$

4. *Velocity*: this is the rate of change of distance with displacement with time. It is a vector quantity. Its unit is metre per seconds (m/s)

$$\text{Velocity} = \frac{\text{displacement}}{\text{Time}}$$

5. *Acceleration*: this is the increasing rate of change of distance with time. It is a vector quantity. Its unit is metre per seconds-square ( $\text{m/s}^2$ ). Retardation or deceleration is a negative acceleration.

$$\text{Acceleration} = \frac{\text{velocity}}{\text{Time}}$$

## EVALUATION I

Sketch the velocity-time graph for a body that starts from rest and accelerates uniformly to a certain velocity. If it maintains this for a given period before its eventual deceleration. Indicate the following:

- 1 Uniform acceleration, retardation
- 2 Total distance travelled

## DERIVATION OF EQUATIONS OF LINEAR MOTION

v= final velocity

u = initial velocity

a = acceleration

t = time

s = distance

Average speed =  $\frac{\text{total distance}}{\text{Time}}$

Total distance= average speed x time

$$s = \frac{(u + v)}{2} \times t \text{ ----- (1)}$$

From the definition of acceleration

$$a = \frac{(v-u)}{t} \text{ ----- (2)}$$

From equation (2) substitute for 't' into equation (1)

$$v^2 = u^2 + 2as \text{ ----- (3)}$$

From equation (2) substitute for 'v' into equation (1)

$$s = ut + \frac{1}{2}(at^2) \text{ ----- (4)}$$

### Calculations Using the Equation of Motion

1. A car moves from rest with an acceleration of  $0.2 \text{ m/s}^2$ . Find its velocity when it has covered distance of 50m

$$u = 0 \text{ m/s}; a = 0.2 \text{ m/s}^2; s = 50 \text{ m}; v = ?$$

$$v^2 = u^2 + 2as$$

$$v^2 = (0)^2 + 2 (0.2 \times 50)$$

$$v^2 = 20$$

$$v = \sqrt{20}$$

$$v = 2\sqrt{5} \text{ m/s}$$

2. A car travels with a uniform velocity of  $108 \text{ km/hr}$ . How far does it travel in  $\frac{1}{2}$  a minute?

### **Solution**

$$v = 108 \text{ km/hr}; t = \frac{1}{2} \text{ minutes}; \text{Distance} = ?$$

$$v = 108 \text{ km/hr} = \frac{108 \times 1000}{3600}$$

$$v = 30 \text{ m/s}$$

$$t = \frac{1}{2} \times 60 = 30 \text{ secs}$$

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

$$\text{Distance} = \text{speed} \times \text{time}$$

$$s = 30 \times 30$$

$$s = 900 \text{ m}$$

### **CLASS ACTIVITY**

(1) A train slows from  $108 \text{ km/hr}$  with a uniform retardation of  $5 \text{ m/s}^2$ . How long will it take to reach  $18 \text{ km/hr}$  and what is the distance covered?

(2) An orange fruit drops to the ground from the top of a tree  $45 \text{ m}$  tall. How long does it take to reach the ground? ( $g = 10 \text{ m/s}^2$ )

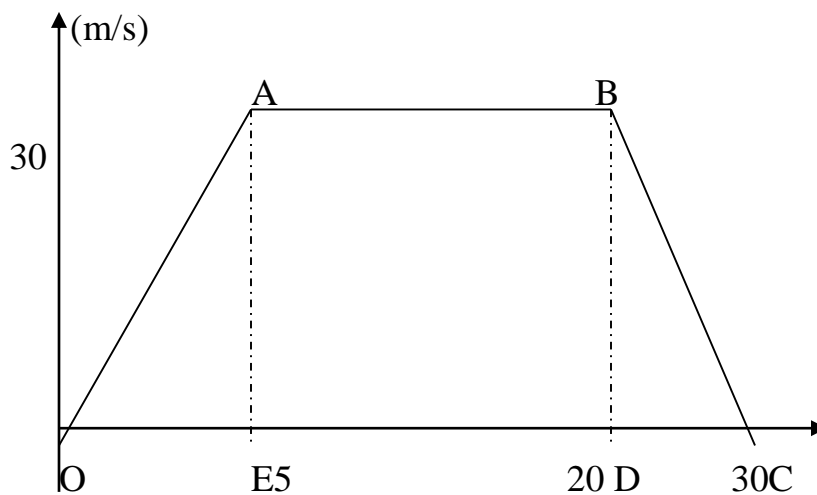
(3) A car moving with a speed of 90 km/h was brought uniformly to rest by the application of brake in 10s. How far did the car travel after the far did the car travel after the brakes were applied .calculate the distance it covers in the last one second its motion.

### FURTHER ACTIVITY

A car starts from rest and accelerates uniformly until it reaches a velocity of 30m/s after 5secs. It travels with this uniform velocity for 15secs and it is then brought to rest in 10secs with a uniform acceleration. Determine:

- (a) The acceleration of the car
- (b) The retardation
- (c) The distance covered after 5secs
- (d) The total distance covered.

Solution



$$\begin{aligned} \text{(a) Acceleration} &= \frac{AE}{EO} = \frac{30}{5} \\ &= \underline{6\text{m/s}^2} \end{aligned}$$

$$\begin{aligned} \text{(b) Retardation} &= \frac{CB}{DC} = \frac{0-30}{10} \\ &= \underline{-3\text{m/s}^2} \end{aligned}$$

(c) The distance covered after 5secs = the area is given by area of the triangle

$$s = \frac{1}{2} b h$$

$$s = \frac{1}{2} (5) 30$$

$$s = 75\text{m}$$

(d) The total distance covered = area of the trapezium OABC

$$s = \frac{1}{2} (AB + OC) \times h$$

$$s = \frac{1}{2} (15 + 30) \times 30$$

$$s = 45 \times 15$$

$$s = 675 \text{ m}$$

## **MOTION UNDER GRAVITY**

A body moving with a uniform acceleration in space does so under the influence of gravity with a constant acceleration . ( $g = 10 \text{ m/s}^2$ ). In dealing with vertical motion under gravity, it must be noted that:

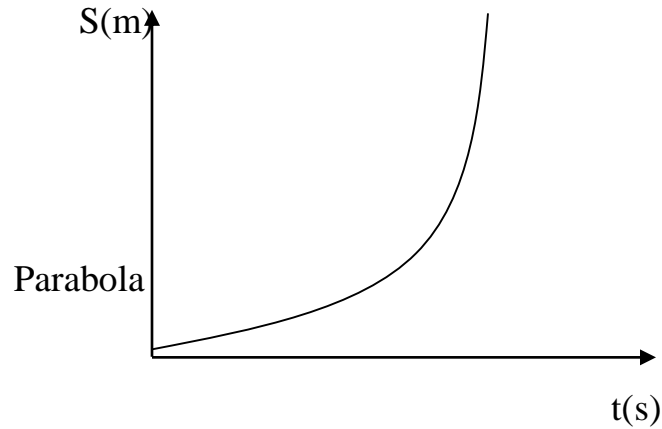
- $a = g$  is positive for a downward motion
- $a = -g$  for an upward motion
- the velocity  $v = 0$  at maximum height for a vertically projected object
- The initial velocity  $u = 0$  for a body dropped from rest above the ground
- For a re-bouncing body the heights above the ground is zero
- The time of fall of two objects of different masses has nothing to do with their masses but is dependent on the distance and acceleration due to gravity as shown below

$$s = ut + \frac{1}{2} gt^2$$

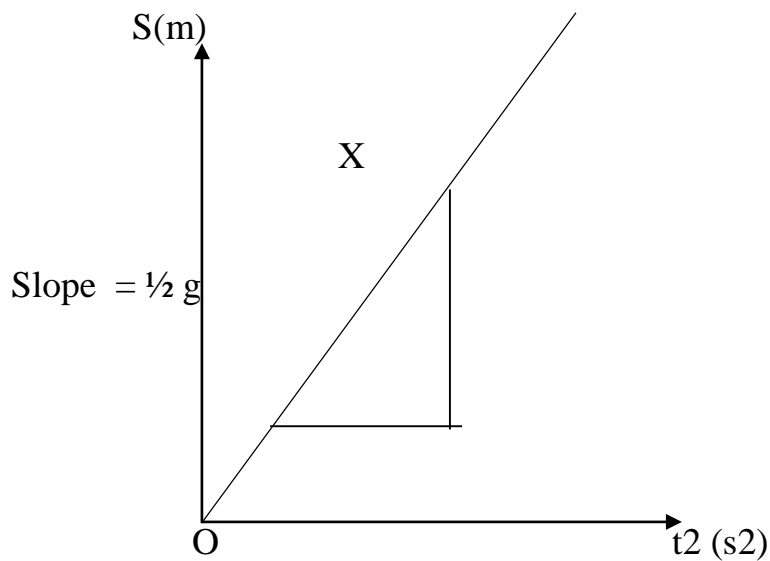
$$s = \frac{1}{2} gt^2 \text{ (} u=0; \text{ initial velocity of an object dropping from a height)}$$

$$t = \sqrt{(2s)/g}$$

The above relationship can also be used to determine the value of acceleration due to gravity. If we plot  $s$  against  $t$ , it will give us a parabolic curve.



But the graph of  $s$  against  $t^2$  will give us a straight line through the origin with slope  $\frac{1}{2}g$  from which  $g$  can be computed



### CALCULATIONS

1. A ball is thrown vertically into the air with an initial velocity,  $u$ . What is the greatest height reached?

#### **Solution**

$$v^2 = u^2 + 2as$$

$$u = u; a = -g; v = 0$$



$$0^2 = u^2 + 2(-g) s$$

$$2gs = u^2$$

$$s = u^2 / 2g$$

2. A ball is released from a height of 20m. Calculate:

(i) the time it takes to fall

(ii) the velocity with which it hits the ground

$$a = +g$$

$$u = 0$$

$$s = 20\text{m}$$

$$t = ?$$

$$t = \sqrt{2s/g}$$

$$t = \sqrt{(2 \times 20 / 10)}$$

$$t = 2\text{secs}$$

$$v = u + gt$$

$$v = gt$$

$$v = 10 \times 2$$

$$v = 20 \text{ m/s}$$

## CLASSWORK

1. Define these parameters (a) acceleration (b) velocity (c) displacement
2. A lorry starts from rest and accelerates uniformly until it reaches a velocity of 50 m/s after 10secs. It travels with uniform velocity for 15secs and is brought to rest in 5secs with a uniform retardation. Calculate:
  - a) The acceleration of the lorry
  - b) The retardation
  - c) The total distance covered
  - d) The average speed of the lorry

## ASSIGNMENT

### SECTION A

1. A body is uniformly retarded comes to rest in 10s after travelling a distance of 20m. Calculate its initial velocity (a) 4.0m/s (b) 2.0m/s (c) 20.0m/s (d) 0.5m/s
2. A body accelerates uniformly rest at the rate of 3m/s for 8 seconds. Calculate the distance it covers. (a) 24m (b) 48m (c) 72m (d) 96m.
3. A particle accelerates uniformly from rest at  $6\text{m/s}^2$  for 8secs and then decelerates uniformly to rest in the next 5 seconds. Determine magnitude of the deceleration (a)  $9.6\text{ m/s}^2$  (b)  $-9.6\text{ m/s}^2$  (c)  $6.9\text{ m/s}^2$  (d)  $-6.9\text{ m/s}^2$
4. A car took off from rest and covered a distance of 80m on a straight road in 10s. Calculate the magnitude of its acceleration (a)  $1.25\text{m/s}^2$  (b)  $1.60\text{ m/s}^2$  (c)  $4.00\text{ m/s}^2$  (d)  $8.0\text{ m/s}^2$
5. An object is released from rest at a height of 20m. Calculate the time it takes to fall to the ground (  $g= 10\text{m/s}^2$ ) (a) 1s (b) 2s (c) 3s (d) 4s.

### SECTION B

1. A ball thrown vertically upwards from the ground level his the ground after 4s. Calculate the maximum height reached during its journey
2. A particle start from rest and moves with constant acceleration of  $0.5\text{m/s}^2$ . Calculate the time taken by the particle to cover a distance of 25m.
3. A car takes off from rest and covers a distance of 80m on a straight road in 10secs .Calculate its acceleration
2. A particle accelerates uniformly from rest at  $6\text{m/s}^2$  for 8secs and then decelerates uniformly to rest in the next 7secs .Determine the magnitude of the deceleration.

## WEEK 4

### PROJECTILES AND ITS APPLICATION

#### CONTENTS

- Meaning of projectile
- Terms associated with projectiles
- Uses of projectile

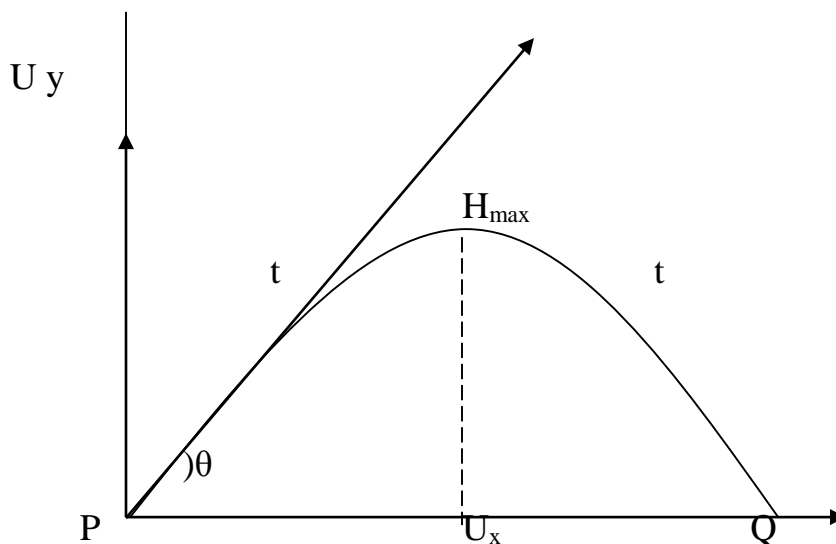
#### MEANING OF PROJECTILE

A projectile motion is one that follows a curved or parabolic path .It is due to two independent motions at right angle to each other .These motions are

- a horizontal constant velocity
- a vertical free fall due to gravity

Examples of projectile motion are the motion of;

- a thrown rubber ball re-bouncing from a wall
- An athlete doing the high jump
- A stone released from a catapult
- A bullet fired from a gun
- A cricket ball thrown against a vertical wall.



$$U_y = U \sin \theta \quad (\text{vertical component}) \quad \text{-----} 1$$

$$U_x = U \cos \theta \quad (\text{horizontal component}) \quad \text{-----} 2$$

## TERMS ASSOCIATED WITH PROJECTILE

1. *Time of flight (T)* - The time of flight of a projectile is the time required for it to return to the same level from which it projected.

$t$  = time to reach the greatest height

$$V = u + at \quad (\text{but, } v = 0, a = -g)$$

$$0 = u \sin \theta - gt$$

$$t = \frac{U \sin \theta}{g} \quad \text{-----} 3$$

$$T = 2t = \frac{2U \sin \theta}{g} \quad \text{-----} 4$$

2. *The maximum height (H)* - is defined as the highest vertical distance reached measured from the horizontal projection plane.

For maximum height H,

$$V^2 = U^2 \sin^2 \theta - 2gH$$

At maximum height H,  $V=0$

$$2gH = \frac{U^2 \sin^2 \theta}{1} \quad \text{-----} 5$$

3. *The range (R)* - is the horizontal distance from the point of projection of a particle to the point where the particle hit the projection plane again.

Horizontally, considering the range covered

Using  $S = ut + \frac{1}{2}at^2$  (where  $a=0$  for the horizontal motion)

OR

$$S = R = U \cos \theta \times t \quad (\text{distance} = \text{velocity} \times \text{time; there time is the time of flight})$$

$$R = \frac{U \cos \theta (2 U \sin \theta)}{g}$$

$$R = \frac{2U^2 \sin \theta \cos \theta}{g}$$

From Trigonometry function

$$2 \sin \theta \cos \theta = \sin 2\theta$$

$$R = \frac{U^2 \sin 2\theta}{g}$$

For maximum range  $\theta = 45^\circ$

$$\sin 2\theta = \sin 2(45) = \sin 90^\circ = 1$$

$$R = \frac{U^2}{g}$$

$$R_{\max} = \frac{U^2}{g}$$

## USE OF PROJECTILES

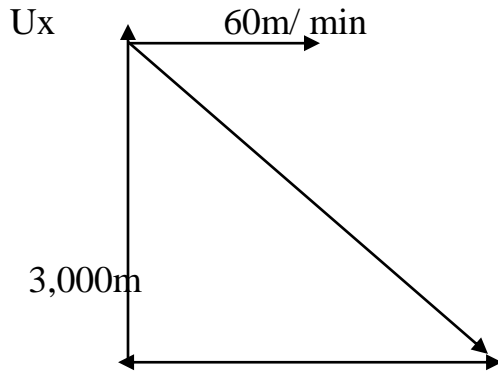
1. To launch missiles in modern warfare
2. To give athletes maximum takeoff speed at meets

In artillery warfare, in order to strike a specified target, the bomb must be released when the target appears at the angle of depression  $\phi$  given by:

$$\tan \phi = \frac{1}{u} \sqrt{gh/2}$$

## EXAMPLES

1. A bomber on a military mission is flying horizontally at a height of 2000 m above the ground at  $60 \text{ km min}^{-1}$ . It drops a bomb on a target on the ground. Determine the acute angle between the vertical and the line joining the bomber and the target at the instant. The bomb is released



Horizontal velocity of bomber =  $60\text{km/min} = 10^3 \text{ ms}^{-1}$

Bomb falls with a vertical acceleration of  $g = 10\text{m/s}^2$

At the release of the bomb, it moves with a horizontal velocity equals that of the aircraft i.e.  $1000\text{m/s}$

Considering the vertical motion of the bomb we have

$$h = ut + \frac{1}{2}gt^2 \quad (u=0)$$

$$h = \frac{1}{2}gt^2$$

Where:  $t$  is the time the bomb takes to reach the ground:  $300 = \frac{1}{2}gt^2$

$$t^2 = 600$$

$$t = 10\sqrt{6} \text{ sec}$$

Considering the horizontal motion we have that horizontal distance moved by the bomb in time  $t$  is given by

$$s = \text{horizontal velocity} \times \text{time}$$

$$s = 1000 \times 10\sqrt{6}$$

$$s = 2.449 \times 10^4 \text{ m}$$

$$\text{But } \tan\theta = \frac{s}{h} = \frac{2.449 \times 10^4}{3,000}$$

$$\theta = 83.02^\circ$$

2. A stone is shot out from a catapult with an initial velocity of  $30\text{m/s}$  at an elevation of  $60^\circ$ . Find

a. the time of flight

- b. the maximum height attained
- c. the range
- a. The time of flight

$$T = \frac{2U \sin \theta}{g}$$

$$g$$

$$T = \frac{2 \times 30 \sin 60^\circ}{10}$$

$$10$$

$$T = 5.2s$$

- b. The maximum height,

$$H = \frac{U^2 \sin^2 \theta}{2g}$$

$$2g$$

$$H = \frac{30^2 \sin^2 (60)}{20}$$

$$20$$

$$\underline{H = 33.75 \text{ m}}$$

- c. The range,

$$R = \frac{U^2 \sin 2\theta}{g}$$

$$g$$

$$R = \frac{30^2 \sin^2 (60)}{10}$$

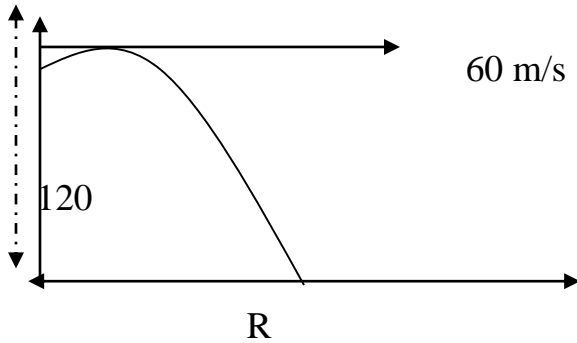
$$10$$

$$R = 90 \sin 120$$

$$R = \underline{77.9 \text{ m}}$$

3. A body is projected horizontally with a velocity of 60m/s from the top of a mast 120m above the ground, calculate

(i) Time of flight, and (ii) Range



a.  $s = ut + \frac{1}{2}gt^2$

$a = g, u = 0$

$120 = \frac{1}{2}(10)t^2$

$t^2 = 24$

$t = 24$

$t = 4.9s$

b.  $\text{Range} = u \cos \theta \times T.$

But in this case  $\theta = 0$

$\cos 0 = 1$

$R = ut$

$R = 60 \times 4.9$

$R = 294m$

4. A stone is projected horizontally with a speed of 10m/s from the top of a tower 50m high and with what speed does the stone strike the ground?

**Solution**

$v^2 = u^2 + 2gh$

$v^2 = 10^2 + (2 \times 10 \times 50)$

$v^2 = 100 + 1000$

$v^2 = 1100$

$v = 33.17m/s$



6. A projectile is fired at an angle of  $60^\circ$  with the horizontal with an initial velocity of  $80\text{m/s}$ . Calculate:

- i. the time of flight
- ii. the maximum height attained and the time taken to reach the height
- iii. the velocity of projection 2 seconds after being fired ( $g = 10\text{m/s}^2$ )

$$\theta = 60^\circ; u = 80\text{m/s}$$

$$\text{i. } T = \frac{2 U \sin \theta}{g}$$

$$T = \frac{2 \times 80 \sin 60^\circ}{10}$$

$$T = 13.86 \text{ s}$$

$$\text{ii. A. } H = \frac{u^2 \sin 2\theta}{2g}$$

$$H = \frac{80 \times 80 \times \sin 60^\circ}{20}$$

$$H = 240 \text{ m}$$

$$\text{B. } t = \frac{U \sin \theta}{g}$$

$$t = \frac{80 \sin 60^\circ}{10}$$

$$t = 6.93 \text{ s}$$

$$R = \frac{U^2 \sin 2\theta}{g}$$

$$R = \frac{80^2 \sin 2(60^\circ)}{10}$$

$$10$$

$$R = 640 \sin 120$$

$$R = 554.3\text{m}$$

$$\text{iii. } V_y = U \sin \theta - gt$$

$$V_y = 80 \sin 60 - 20$$

$$V_y = \underline{49.28\text{m/s}}$$

$$U_x = U \cos \theta$$

$$U_x = 80 \cos 60$$

$$U_x = 40 \text{ m/s}$$

$$U^2 = U_y^2 + U_x^2$$

$$U^2 = 49.28^2 + 40^2$$

$$U = \sqrt{(1600 + 2420)}$$

$$U = 63.41 \text{ m/s}$$

## CLASSWORK

1. (a) Define the term projectile (b) mention two application of projectiles
2. A ball is projected horizontally from the top of a hill with a velocity of 30m/s. if it reaches the ground 5 seconds later, the height of the hill is
3. A stone propelled from a catapult with a speed of 50m/s attains a height of 100m. Calculate: a. the time of flight b. the angle of projection c. the range attained.

## ASSIGNMENT

### SECTION A

1. A stone is projected at an angle 60 and an initial velocity of 20m/s determine the time of flight (a) 34.6s (b) 3.46s (c) 1.73s (d) 17.3s (e) 6.92s

2. A stone is projected at an angle  $60^\circ$  and an initial velocity of 20m/s determine the time of flight (a) 34.6s (b) 3.46s (c) 1.73s (d) 17.3s (e) 6.92s
3. For a projectile the maximum range is obtained when the angle of projection is; (a)  $60^\circ$  (b)  $30^\circ$  (c)  $45^\circ$  (d)  $90^\circ$
4. The maximum height of a projectile projected with an angle of  $\theta$  to the horizontal and an initial velocity of U is given by  
 (a)  $\frac{U \sin^2 \theta}{g}$  (b)  $\frac{U^2 \sin^2 \theta}{2g}$  (c)  $\frac{U^2 \sin \theta}{g}$  (d)  $\frac{2U^2 \sin^2 \theta}{g}$

Use this information to answer questions 5 and 6: An arrow is shot into space with a speed of 125m/s at an angle of  $15^\circ$  to the level ground. Calculate the:

5. Time of flight (a) 5seconds (b) 6.47seconds (c) 16.01seconds (d) 4.7seconds
6. Range of the arrow (a) 350m (b) 781.25m (c) 900m (d) 250.71

## SECTION B

1. A gun fires a shell at an angle of elevation of  $30^\circ$  with a velocity of 2x10m. What are the horizontal and vertical components of the velocity? What is the range of the shell? How high will it rise?
2. (a) What is meant by the range of a projectile? (b) An object is projected into the air with a speed of 50m/s at an angle of  $30^\circ$  above the ground level. Calculate the maximum height attained by the object

## WEEK 5

### NEWTON'S LAW OF MOTION

#### CONTENTS

- Newton's laws of motion
- Conservation of linear momentum
- Collision

#### NEWTON'S LAWS OF MOTION

*Newton's first law of motion* states that everybody continues in its state of rest or of uniform motion in straight line unless it is acted upon by a force. The tendency of a body to remain at rest or, if moving, to continue its motion in a straight line is called the *inertia*. That is why Newton's first law is otherwise referred to as the law of inertia.

*Newton's second law of motion* states that the rate of change of momentum of a body is directly proportional to the applied force and takes place in the direction in of the force.

$$F \propto \frac{mv - mu}{t}$$

$$F \propto \frac{m(v - u)}{t}$$

$$F \propto ma$$

$$F = kma$$

$$\text{Where } k = 1$$

$$F = ma$$

#### MOMENTUM

*Momentum* of a body is the product of the mass and velocity of the body.

The S.I. unit of momentum is kgm/s.

## IMPULSE

*Impulse* is the product of a force and time. It is also defined as the change in momentum. Thus both momentum and impulse have 'Ns' as unit

$$F = m (v-u)/t$$

$$Ft = mv - mu \text{ (where 'mv-mu' is the change of momentum)}$$

$$F \times t = I \text{ (Ns)}$$

*Newton's third law of motion* states that to every action, there is an equal but opposite reaction. A practical demonstration of this law can be observed when a bullet is fired from a gun, the person holding it experiences the backward recoil force of the gun (reaction) which is equal to the propulsive force (action) acting on the bullet.

According to Newton second law of motion, force is proportional to change in momentum

Therefore the momentum of the bullet is equal and opposite to the momentum of the gun i.e.

Mass of bullet x muzzle velocity = mass of gun x recoil velocity

Hence, if:  $m$  = mass of bullet,  $v$  = velocity of bullet,  $M$  = mass of gun,  $V$  = velocity of the recoil of the gun.

Then, the velocity,  $V$ , of the recoil of the gun is given by:

$$MV = mv$$

$$V = mv/M$$

## CONSERVATION OF LINEAR MOMENTUM

The principle of conservation of linear momentum states that *when two or more bodies collide, their momentum remain constant provided there is no external force acting on the system*. This implies that in a closed or isolated system where there is no external force, the total momentum after collision remains constant. The principle is true for both elastic and inelastic collision.

## COLLISION

There are two types of collision- elastic and inelastic.

*In elastic collision* the two bodies collide and then move with different velocities. Both *momentum and kinetic energy are conserved* e.g. collision between gaseous particles, a ball which rebounds to its original height etc.

If the two colliding bodies have masses  $m_1$  and  $m_2$  initial velocities  $u_1$  and  $u_2$  and final velocities  $v_1$  and  $v_2$ . The conservation principle can be mathematically expressed as:

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

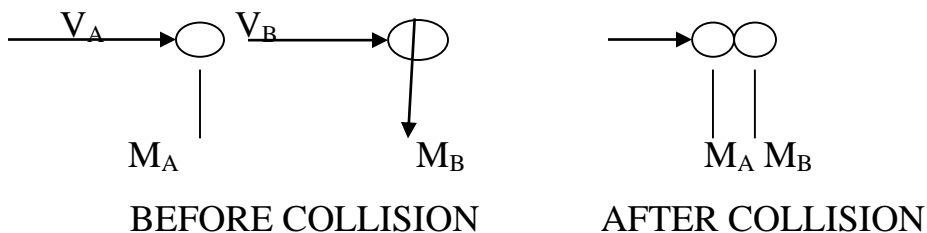
*In an inelastic collision*, the two bodies join together after the collision and with the same velocity. Here, *momentum is conserved but kinetic energy is not conserved* because part of it has been converted to heat or sound energy, leading to deformation.

Thus, the conservation principle can be re-written as:

$$m_1u_1 + m_2u_2 = v(m_1 + m_2)$$

Since momentum is a vector quantity, all the velocities must be measured in the same direction, assigning positive signs to the forward velocities and negative signs to the backward or opposite velocities

### TWO BODIES MOVING IN THE SAME DIRECTION BEFORE COLLISION

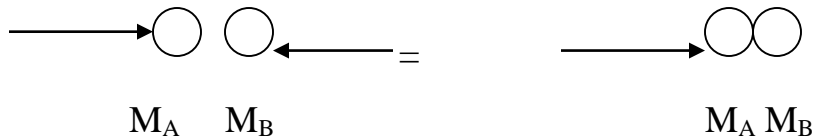


$$M_A V_A + M_B V_B = V(M_A + M_B)$$

$V$  = common velocity

$$V = \frac{M_A V_A + M_B V_B}{(M_A + M_B)}$$

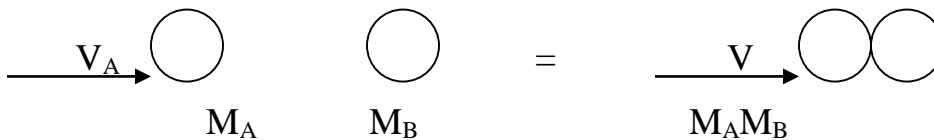
## TWO BODIES TRAVELLING IN OPPOSITE DIRECTION



$$M_A V_A - M_B V_B = V (M_A + M_B)$$

$$V = \frac{M_A V_A + M_B V_B}{M_A + M_B}$$

## COLLISION BETWEEN A STATIONARY AND MOVING BODY



The momentum of a stationary body is zero because velocity is zero

$$M_A V_A + 0 = V (M_A + M_B)$$

$$V = \frac{M_A V_A}{M_A + M_B}$$

### EXAMPLE

- Two moving toys of masses 50kg and 30kg are traveling on the same plane with speeds of 5 m/s and 3 m/s respectively in the same direction. If they collide and stick together, calculate their common velocity.

$$M_A V_A + M_B V_B = V (M_A + M_B)$$

$$V = \frac{M_A V_A + M_B V_B}{(M_A + M_B)}$$

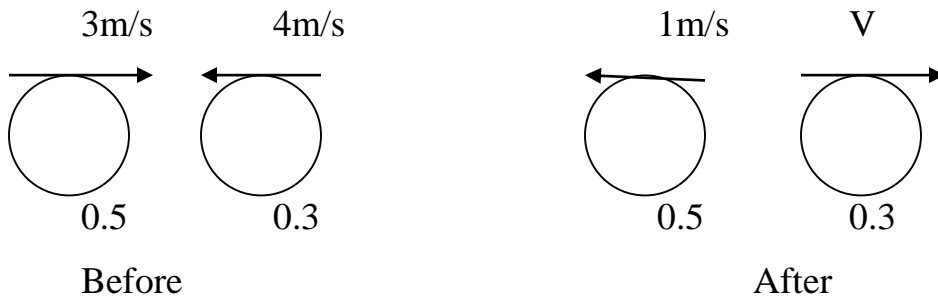
$$V = \frac{(50 \times 5) + (30 \times 3)}{50 + 30}$$

$$V = \frac{250 + 90}{80}$$

$$V = \frac{340}{80}$$

$$V = 4.05 \text{ m/s}$$

2. Two balls of masses 0.5 kg and 0.3kg move towards each other in the same line at speeds of 3 m/s and 4 m/s respectively. After the collision, the first ball has a speed of 1m/s in the opposite direction. What is the speed of the second ball after collision



$$3 \times 0.5 + (0.3 \times -4) = 0.5 (-1) + 0.3v$$

$$1.5 - 1.2 = -0.5 + 0.3v$$

$$0.3v = 2.0 - 1.2$$

$$V = 0.8 / 0.3$$

$$V = 2.7 \text{ m/s}$$

3. A gun of mass 100kg fires a bullet of mass 20g at a speed of 400m/s. What is the recoil velocity of the gun?

### **Solution**

Momentum gun = momentum of bullet

$$MV = m v$$

$$10 \times V = 0.002 \times 400$$

$$V = \frac{0.002 \times 400}{10}$$

$$10$$



$$V = 0.8 \text{ m/s}$$

## CLASSWORK

1. Derive from Newton's law the relationship between Force, mass and acceleration
2. State Newton laws of motion and explain the consequences of each law
3. State the principle of conservation of linear momentum.
4. A 15kg monkey hangs from a cord suspended from the ceiling of an elevator. The cord can withstand a tension of 200N and breaks as the elevator accelerates. What was the elevators minimum acceleration ( $g=10\text{m/s}^2$ ).

## ASSIGNMENT

### SECTION A

1. A force acts on a body for 0.5s changing its momentum from  $16\text{kgms}^{-1}$  to  $21\text{kgms}^{-1}$ . Calculate the magnitude of the force (a) 42N (b) 37N (c) 32N (d) 10N
2. A ball of mass 6kg moving with a velocity of 10m/s collides with a 2kg ball moving in the opposite direction with a velocity of 5m/s. After the collision the two balls coalesce and move in the same direction. Calculate the velocity of the composite body (a) 5m/s (b) 6.25m/s (c) 8.75m/s (d) 12m/s
3. A machine gun with mass of kg fires a 50g bullet at a speed of 100m/s. The recoil speed of the machine gun is (a) 0.5m/s (b) 1.0m/s (c) 1.5m/s (d) 2.0m/s
4. When taking a penalty kick, a footballer applies a force of 30N for a period of 0.05S. If the mass of the ball is 0.075kg, calculate the speed with which the ball moves off (a) 4.5m/s (b) 11.25m/s (c) 20m/s (d) 45m/s

5. A jet engine develops a thrust of 270N when the velocity of the exhaust gases relative to the engine is 300m/s, what is the mass of the gas ejected per second? (a) 81.00kg (b) 9.00kg (c) 0.90kg (d) 0.09kg

## **SECTION B**

1. An object of mass 5kg slides down a smooth plane at an angle of  $25^\circ$ . If the object starts from rest, find; (i) its velocity after 3m (ii) its momentum 3m from the starting point (iii) the force causing it moves ( $g=10\text{m/s}^2$ )
2. State the law of conservation of linear momentum. A 3kg rifle lies on a smooth table when it suddenly discharges, firing a bullet of 0.02kg with a speed of 500m/s. Calculate the recoil speed of the gun.
3. A bullet of mass 120g is fired horizontally into a fixed wooden block with a speed of 20m/s. The bullet is brought to rest in the block in 0.1s by a constant resistance. Calculate the: (i) magnitude of the resistance (ii) the distance moved by the bullet in the wood

## WEEKS 6&7

### EQUILIBRIUM OF FORCES

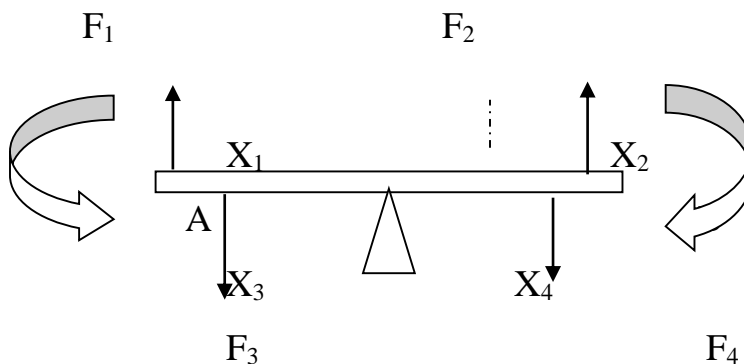
#### CONTENTS

- Conditions for equilibrium
- Principles of moment
- Conditions for equilibrium of a rigid body

#### CONDITIONS FOR EQUILIBRIUM

A body is said to be in equilibrium if under the action of several forces, it does not accelerate or rotate.

1. The sum of the upward forces must be equal to the sum of the downward forces.
2. The sum of the clockwise moment about a point must be equal to the sum of the anticlockwise moment about the same point



$$F_1 + F_2 = F_3 + F_4$$

$$(F_1 + F_2) - (F_3 + F_4) = 0$$

$$\text{Clockwise moment} = F_2 X_2 + F_4 X_4$$

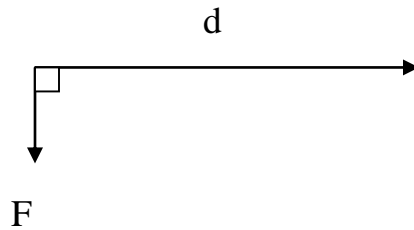
$$\text{Anticlockwise moment} = F_1 X_1 + F_3 X_3$$

$$(F_1 X_1 + F_3 X_3) - (F_2 X_2 + F_4 X_4) = 0$$

$$\text{Sum of clockwise moment} = \text{sum of anticlockwise moment}$$

## MOMENT OF A FORCE

The moment of a force is the product of the force and the perpendicular distance

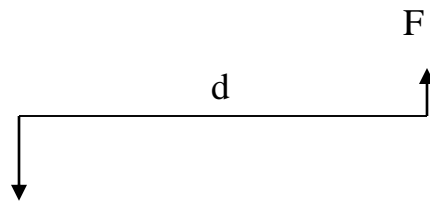


$$M = F \times \text{distance}$$

$$\text{Unit} = \text{Nm}$$

## COUPLE

A couple is a system of two parallel, equal and opposite forces acting along the same line



The moment of a couple is the product of one of the forces and the perpendicular distance between the lines of action of the two forces

$$M = f \times 2r$$

$$M = f \times d$$

The distance between the two equal forces is called the *arm of the couple*

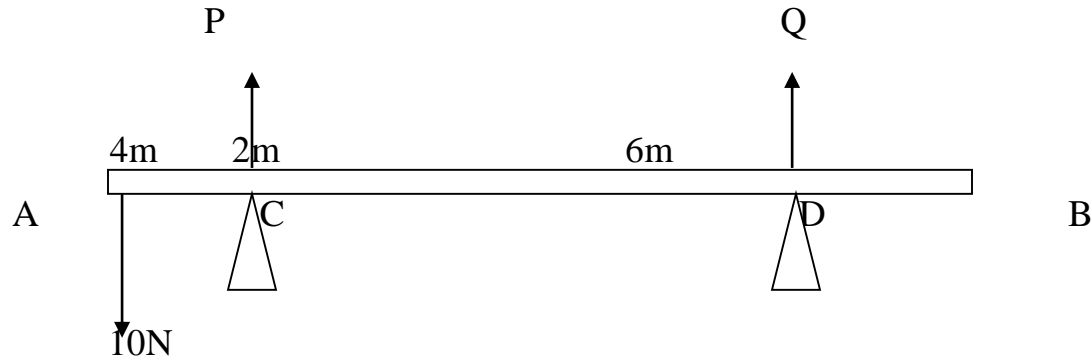
The moment of a couple is also called a *torque*

### Application of the Effect of Couples

1. It is easier to turn a tap on or off by applying couple
2. It is easier to turn a steering wheel of a vehicle by applying a couple with our two hands instead of a single force with one arm.

## EXAMPLES

1. A light beam AB sits on two pivots C and D. A load of 10N hangs at 0; 2m from the support at C. Find the value of the reaction forces P and Q at C and D respectively.



$$P + Q = 10N$$

$$10 \times 2 = Q (2 + 6)$$

$$20 = 8Q$$

$$Q = 20/8 = 2.5 \text{ N}$$

OR

Taking moment about D

$$P \times 8 = 10 \times 6$$

$$P = 60/8$$

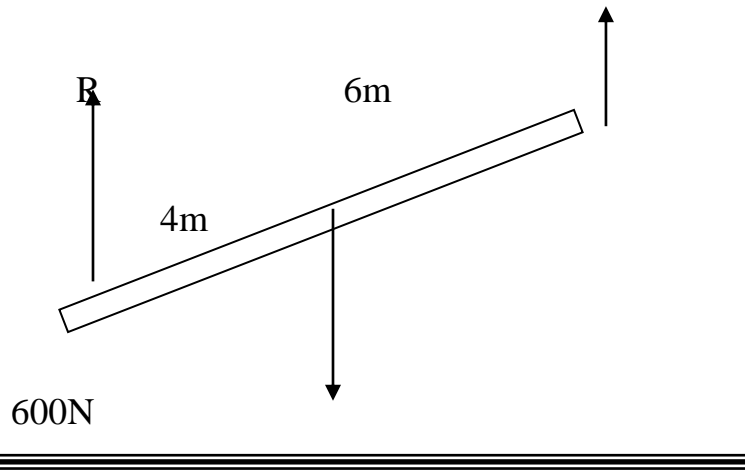
$$P = 7.5N$$

$$Q = 10 - 7.5$$

$$Q = 2.5 \text{ N}$$

2. A pole AB of length 10m and weight 600N has its center of gravity 4m from the end A, and lies on horizontal ground. Draw a diagram to show the forces acting on the pole when the end B is lifted. Prove that this force applied at the end A will not be sufficient to lift the end A from the ground.

P



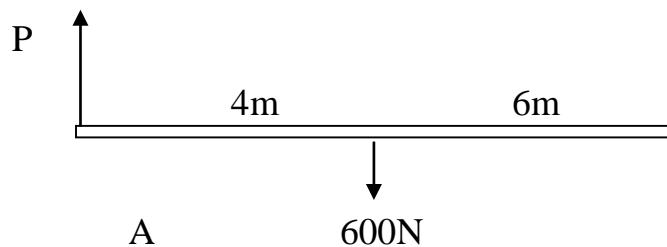
Clockwise moment  $= 600 \times 4 = 2400 \text{ Nm}$

Anticlockwise moment  $= p \times 10 = 10p \text{ Nm}$

$$P = 240 \text{ Nm}$$

If this force of 240Nm is applied at A, we have

$$P = 240 \text{ Nm}$$



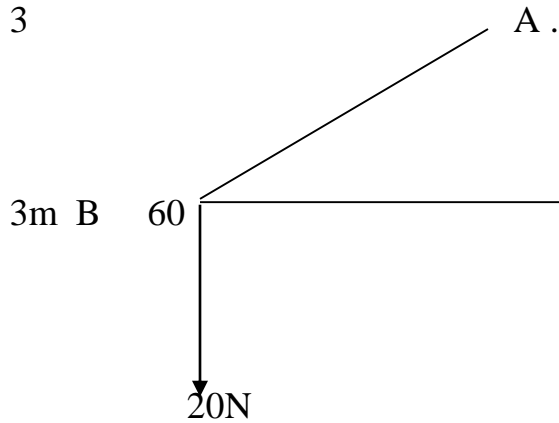
Taking moment about B, we have

Clockwise moment  $= 240 \times 10 = 2400 \text{ Nm}$

Anticlockwise moment  $= 600 \times 6 = 3600 \text{ Nm}$

The anticlockwise moment is greater than the clockwise moment.

Therefore, the 240N force A will not be sufficient to lift the end A because the turning effect due to the 600N force far exceeds that due to the 240N force



Find the moment of the force of 20N in the diagram above about A and B

Taking moment about A

$$\cos 60 = d/3m$$

$$D = 3 \cos 60$$

$$D = 1.5m$$

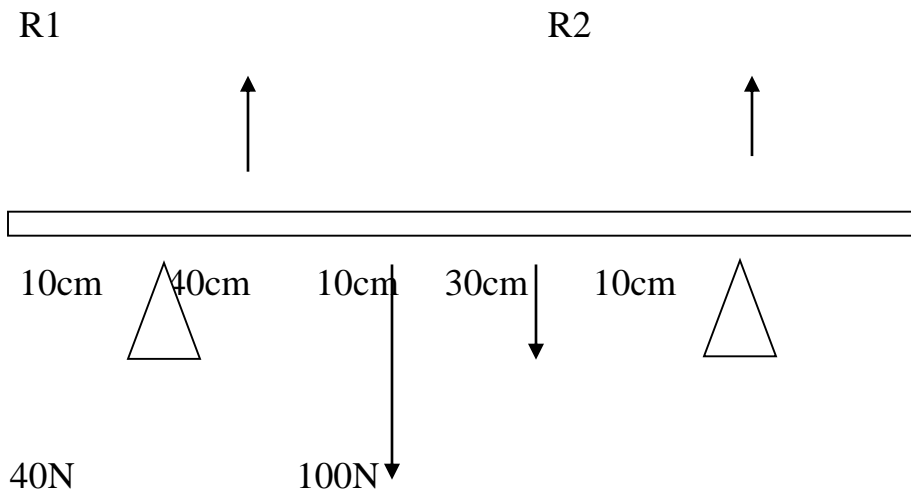
$$\text{Moment about A} = F \times d$$

$$M = 20 \times 1.5$$

$$= 30 \text{ Nm}$$

The Moment about B = 0

3. A uniform rod 1m long weighing 100N is supported horizontally on two knife edges placed 10cm from its ends. What will be the reaction at the support when a 40N load is suspended 10cm from the midpoint of the rod.



$$R_1 + R_2 = 140\text{N}$$

Taking moment about  $R_1$

$$R_2 \times 80 = (100 \times 40) + (40 \times 50)$$

$$80R_2 = 4000 + 2000$$

$$R_2 = 6000/80$$

$$R_2 = 75\text{N}$$

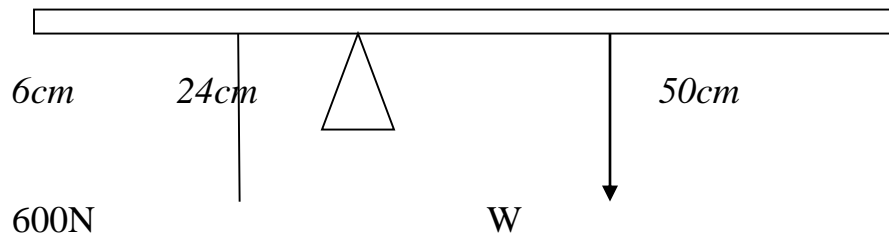
$$R_1 = 140 - 75$$

$$= 65\text{N}$$

4. A metre rule is found to balance horizontally at the 50cm mark. When a body of mass 60kg is suspended at the 6cm mark, the balance point is found to be at the 30cm mark, calculate:

-The weight of the metre rule

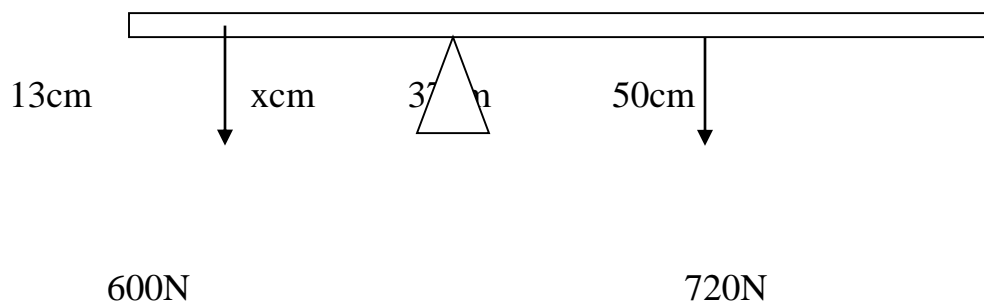
-The distances of the balance point to the 60kg mass if the mass is moved to the 13cm mark



$$w \times 20 = 24 \times 600$$

$$w = 14400/20$$

$$= 720\text{N}$$





$$600x(X)=720(37-X)$$

$$600x = 6640 - 720x$$

$$600x + 720x = 6640$$

$$x = 6640/1320$$

$$x = 20.18\text{cm}$$

## **CENTRE OF GRAVITY**

The centre of gravity of a body is the point through which the line of action of the weight of the body always passes irrespective of the position of the body. It is also the point at which the entire weight of the body appears to be concentrated.

The centre of mass of a body is the point at which the total mass of the body appears to be concentrated. Sometimes, the center of mass may coincides with the centre of gravity for small objects.

## **STABILITY OF OBJECTS**

There are three types of equilibrium- stable equilibrium, unstable equilibrium, and neutral equilibrium.

1. **Stable equilibrium:** a body is said to be in stable equilibrium if it tends to return to its original position when slightly displaced. A low centre of gravity and wide base will put objects in stable equilibrium e.g. a cone resting on its base ; a racing car with low C.G and wide base; a ball or a sphere in the middle of a bowl.
2. **Unstable equilibrium:** a body is said to be in an unstable equilibrium if when slightly displaced it tends to move further away from its original position e.g. a cone or an egg resting on its apex. High C.G. and a narrow base usually causes unstable equilibrium.

3. Neutral equilibrium: a body is said to be in neutral equilibrium if when slightly displaced, it tends to come to rest in its new position e.g a cone or cylinder or an egg resting on its side.

### **CLASSWORK**

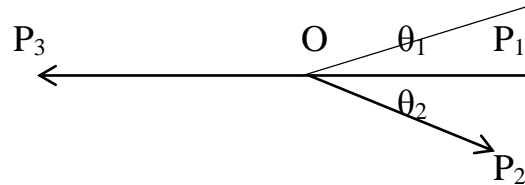
1. When is a body said to be in equilibrium?
2. What is moment?
3. Write short note on the three types of equilibrium

### **ASSIGNMENT**

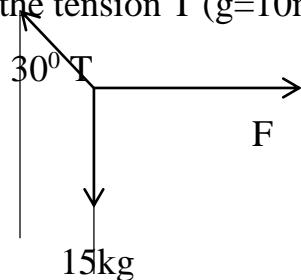
#### **SECTION A**

1. Two forces A and B act at a point at right angles. If their resultant is 50N and their sum is 70N, their magnitudes are: (a) 50N and 20N (b) 20N and 40N (c) 40N and 30N (d) 60N and 10N
2. A uniform metre rule of mass 100g balances at the 40cm mark when a mass X is placed at the 10cm mark. What is the value of X? (a) 33.33g (b) 43.33g (c) 53.33g (d) 63.33g
3. The equilibrant of a system of forces is (a) equal and opposite to the resultant of the forces (b) the force which has the same effect as the system (c) equal to resultant of the system (d) the force that makes the system unstable
4. Two forces forming a couple are separated by a distance of 25cm, if one of the forces equal 40N, what is the moment of the couple? (a) 1000Nm (b) 500Nm (c) 10Nm (d) 5Nm
5. Two forces each of magnitude 10N acts in opposite directions at the end of a table. If the length of the table is 50cm. Find the moment of the couple on the table (a) 0.5Nm (b) 5Nm (c) 50Nm

6. A pole AB of length 5M and weigh 300N has its centre of gravity 2.0M from the end A, and lies on horizontal ground. Calculate the force required to begin to lift this end. (a) 60N (b) 120N (c) 240N
7. Consider the three forces acting at a point O and is in equilibrium as shown below. Which of the equations is/are correct?



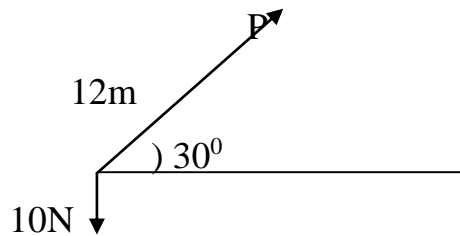
- i.  $P_1 \cos \theta_1 = P_1 \cos \theta_2$       ii.  $P_3 = P_1 \cos \theta_1 + P_2 \cos \theta_2$       iii.  $P_1 \sin \theta_1 = P_2 \cos \theta_2$       (a) I only (b) II only (c) III only (d) II and III only
8. When a body is acted upon by several forces and it does not accelerate or rotates, the body is said to be in (a) space (b) equilibrium (C) motion
9. Two masses 40g and 60g respectively are attached firmly to the ends of a light metre rule. The centre of gravity of the system is: (a) at the midpoint of the metre rule (b) 40cm from the lighter mass (c) 40cm from the heavier mass (d) 60cm from the heavier mass
10. A 50kg mass, suspended from a ceiling is pulled aside with a horizontal force, F, as shown in the diagram below. Calculate the value of the tension T ( $g=10\text{ms}^{-2}$ )



- (a) 300.0N (b) 173.2N (c) 30.0N (d) 17.3

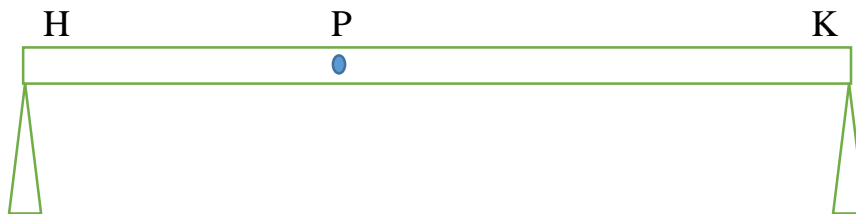
## SECTION B

1. (a) Explain with the aid of diagram what is meant by the moment of a force about a point (b) State the conditions of equilibrium for a number of coplanar parallel forces (c) A metre rule is found to balance at 48cm mark. When a body of mass 60g is suspended at 6cm mark, the balance point is found to be at the 30cm mark. Calculate the: (i) mass of the rule (ii) the distance of the balance point from the zero end if the body were moved to the 13cm mark
2. State the conditions necessary for a body to be in equilibrium, mention the three types of equilibrium with at least two examples each.
- 3.



Use the diagram above to calculate the moment of the force of 10N about the point P

4. A uniform beam HK of length 10m and weighing 200N is supported at both ends as shown below. A man weighing 100N stands at a point P on the beam. If the reactions at H and K are respectively 800N and 400N, then the distance HP is



# MIDTERM PROJECT

*Make paper model of the  
three types of equilibrium*

## WEEKS 8 & 9

### SIMPLE HARMONIC MOTION

#### CONTENT

- Definition
- Speed
- Amplitude
- Displacement
- Acceleration
- Period
- Frequency

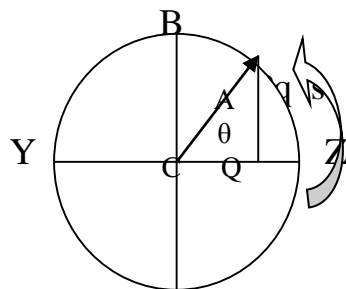
#### DEFINITION

This is the periodic motion of a body or particle along a straight line such that the acceleration of the body is directed towards a fixed point.

A particle undergoing simple harmonic motion will move to and fro in a straight line under the influence of a force. This influential force is called a restoring force as it always directs the particle back to its equilibrium position.

Examples of simple harmonic motions are

- i. loaded test tube in a liquid
- ii Mass on a string
- iii The simple pendulum



As the particle P moves round the circle once, it sweeps through an angle  $\theta = 360$  (or  $2\pi$  radians) in the time T the period of motion. The rate of change of the angle  $\theta$  with time (t) is known as the angular velocity  $\omega$

Angular velocity ( $\omega$ ) is defined by

$$\omega = \frac{\text{angle turned through by the body}}{\text{Time taken}}$$

Time taken

$$\omega = \theta / t \text{ (rad /sec)}$$

$$\theta = \omega t$$

This is similar to the relation distance = uniform velocity x time ( $s = vt$ ) for motion in a straight line

As the angle is changing with time so is the arc length

$$S = r\theta$$

Changing with time. By definition  $\theta$  in radians =  $s/r$  and hence

$$S = r\theta$$

$A = r$  = radius of the circle

$$s/t = r\theta / t = s/r / t$$

$$s/t = s/t \times 1/r = r \theta / t$$

$$v = r \omega$$

The linear velocity  $v$  at any point ,Q whose distance from C the central point is  $x$  is given by

$$V = \omega \sqrt{A^2 - X^2}$$

The minimum velocity , $V_m$  corresponds to the point at  $X = 0$  that is the velocity at the central point or centre of motion .

$$\text{Hence, } V_m = \omega A$$

Thus the maximum velocity of the SHM occurs at the centre of the motion ( $X=0$ ) while the minimum velocity occurs at the extreme position of motion ( $x=A$ ).

## RELATIONSHIP BETWEEN LINEAR ACCELERATION AND ANGULAR VELOCITY

$$X = A \cos \theta$$

$$\theta = \omega t$$

$$X = A \cos \omega t$$

$$\frac{dx}{dt} = -\omega A \sin \omega t$$

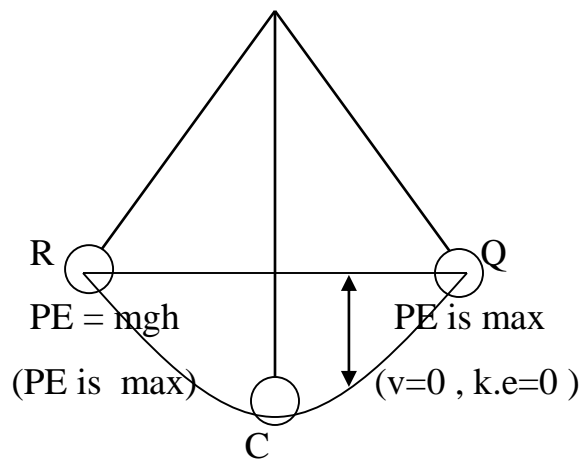
$$\frac{dv}{dt} = -\omega^2 A \cos \omega t$$

$$= -\omega^2 X$$

The negative sign indicates that the acceleration is always inwards towards C while the displacement is measured outwards from C.

- Energy of simple harmonic motion
- Forced vibration and resonance

## ENERGY OF SIMPLE HARMONIC MOTION



$$h=0, PE =0; KE = \frac{1}{2} MV^2; KE \text{ is max}$$



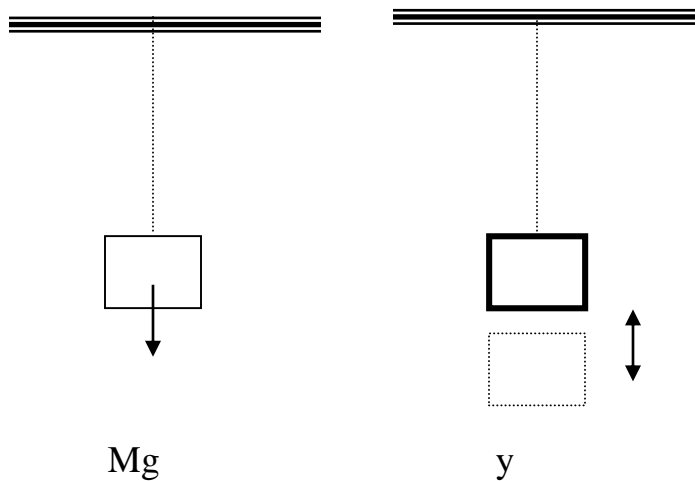
Since force and displacement are involved, it follows that work and energy are involved in simple harmonic motion.

At any instant of the motion, the system may contain some energy as kinetic energy (KE) or potential energy (PE). The total energy (KE + PE) for a body performing SHM is always conserved although it may change form between PE and KE.

When a mass is suspended from the end of a spring stretched vertically downwards and released, it oscillates in a simple harmonic motion. During this motion, the force tending to restore the spring to its elastic restoring force is simply the elastic restoring force which is given by

$$F = -ky$$

K is the force constant of the spring



The total work done in stretching the spring at distance y is given by

$$W = \text{average force} \times \text{displacement}$$

$$W = \frac{1}{2} ky \times y = \frac{1}{2} ky^2$$

Thus the maximum energy total energy stored in the spring is given by

$$W = \frac{1}{2} KA^2$$

A = amplitude (maximum displacement from equilibrium position).

This maximum energy is conserved throughout the motion of the system.

At any stage of the oscillation, the total energy is

$$W = \frac{1}{2} KA^2$$

$$W = \frac{1}{2} mv^2 + \frac{1}{2} ky^2$$

$$\frac{1}{2} mv^2 = \frac{1}{2} KA^2 - \frac{1}{2} ky^2$$

$$v^2 = \frac{k}{m} (A^2 - y^2)$$

$$V = \sqrt{\frac{k}{m}(A^2 - y^2)}$$

The constant K is obtained from

Hooke's law in which

$$F = mg = ke$$

Where e is the extension produced in the spring by a mass m

$$\text{But } V = \omega \sqrt{A^2 - X^2}$$

$$\text{Therefore } \omega = \sqrt{k/m}$$

$$\text{Hence the period, } T = 2\pi/\omega$$

$$T = 2\pi\sqrt{m/k}$$

### EXAMPLE

A body of mass 20g is suspended from the end of a spiral spring whose force constant is  $0.4\text{Nm}^{-1}$ . The body is set into a simple harmonic motion with amplitude 0.2m. Calculate:

- The period of the motion
- The frequency of the motion
- The angular speed
- The total energy
- The maximum velocity of the motion
- The maximum acceleration

### SOLUTION

- $T = 2\pi \sqrt{m/k} = 2\pi \sqrt{0.02/0.4} = 0.447 \pi \text{ sec} = 1.41 \text{ sec}$
- $f = 1/T = 1/1.41 = 0.71\text{Hz}$

c.  $\omega = 2\pi f = 2\pi \times 0.71 = 4.46 \text{ rad. S}^{-1}$

d. Total energy =  $\frac{1}{2} KA^2 = \frac{1}{2} (0.4) (0.2)^2 = 0.008 \text{ J}$

e.  $\frac{1}{2} mv^2 = \frac{1}{2} KA^2$

$$Vm^2 = \frac{0.008 \times 2}{0.02}$$

$$= 0.8$$

$$Vm = 0.89 \text{ m/s}$$

Or  $V = \omega A$

$$= 4.46^2 \times 0.2$$

$$= 3.98 \text{ m/s}^2$$

## FORCED VIBRATION AND RESONANCE

Vibrations resulting from the action of an external periodic force on an oscillating body are called forced vibrations. Every vibrating object possesses a natural frequency ( $f_0$ ) of vibration. This is the frequency with which the object will oscillate when it is left undisturbed after being set into vibration. The principle of the sounding board of a piano or the diaphragm of a loudspeaker is based on the phenomenon of forced vibrations.

Whenever the frequency of vibrating body acting on a system coincides with the natural frequency of the system, then the system is set into vibration with relatively large amplitude. This phenomenon is called resonance.

## CLASSWORK

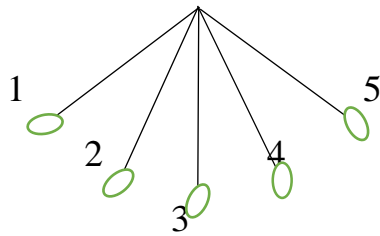
1. What is simple harmonic motion? Give four examples to illustrate simple harmonic motion
2. A particle moves round a circle of radius 10cm with a constant velocity of 20m/s, calculate the angular velocity
3. A particle undergoes simple harmonic motion with an amplitude of 5cm and an angular velocity of  $10\pi \text{ rads}^{-1}$ , calculate: (i) the maximum velocity

- (ii) the velocity when it is 2cm from the equilibrium position (iii) the maximum acceleration of the particle (iv) the period of oscillation
4. Define the following terms: frequency, period, amplitude of simple harmonic motion. What is the relation between period and frequency?

## ASSIGNMENT

### SECTION A

- The period of oscillation of a simple pendulum is 2.0s. Calculate the period if the length of the pendulum is doubled (a) 1.0s (b) 1.4s (c) 2.8s (d) 4.0s
- The period of a body performing simple harmonic motion is 2.0s. If the amplitude of the motion is 3.5cm, calculate the maximum speed of the body ( $\pi=22/7$ ) (a) 22.0cms<sup>-1</sup> (b) 11.0cms<sup>-1</sup> (c) 7.0cms<sup>-1</sup> (d) 1.8cms<sup>-1</sup>
- A pendulum bob, executing simple harmonic motion has 2cm and 12Hz as amplitude and frequency respectively. Calculate the period of the motion (a) 2.00s (b) 0.83s (c) 0.08s (d) 0.06s
- What is the angular speed of a body vibrating at 50 cycles per seconds (a) 200 $\pi$ rads<sup>-1</sup> (b) 400  $\pi$ rads<sup>-1</sup> (c) 100  $\pi$ rads<sup>-1</sup> (d) 50  $\pi$ rads<sup>-1</sup>
- In the diagram below, the maximum potential energy of the swinging pendulum occurs at position(s) (a) 1 and 5 (b) 2 and 4 (c) 3 Only (d) 5 and 3



- The motion of a body is simple harmonic if the: (a) acceleration is always directed towards a fixed point (b) path of motion is a straight line (c) acceleration is directed towards a fixed point and proportional to its distance from the point (d) acceleration is proportional to the square of the distance from a fixed point

7. Which of the following assumptions is made in a simple pendulum experiment? The (a) suspending string is inextensible (b) bob has a finite size (c) bob has a definite mass (d) initial angle of oscillation must be large
8. A simple pendulum has a period of 17.0s. When the length is shortened by 1.5m, its period is 8.5s. Calculate the original length of the pendulum (a) 1.5m (b) 2.0m (c) 3.0m (d) 4.0m
9. The period of oscillatory motion is defined as the (a) average of the time used in completing different numbers of oscillations (b) time to complete a number of oscillations (c) time to complete one oscillation (d) time taken to move from one extreme position to another
10. Which of the following correctly gives the relationship between linear speed  $V$  and angular speed  $\omega$  of a body moving uniformly in a circle of radius  $r$ ? (a)  $v = \omega r$  (b)  $v = \omega^2 r$  (c)  $v = \omega r^2$  (d)  $v^2 = \omega r$

## SECTION B

1. Derive the expression  $T = 2\pi \sqrt{\frac{l}{g}}$  of the period of a simple pendulum. A simple pendulum has a period of 3.45 seconds. When the length of the pendulum is shortened by 1.0m, the period is 2.81 seconds. Calculate: (i) the original length of the pendulum (b) the value of the acceleration due to gravity.
2. A body of mass 0.5kg is attached to the end of a spring and the mass pulled down a distance 0.01m. Calculate: (i) the period of oscillation (ii) the maximum kinetic energy of mass (iii) kinetic and potential energy of the spring when the body is 0.04m below its centre of oscillation. ( $k=50\text{Nm}$ ).

3. A body of mass 0.2kg is executing simple harmonic motion with amplitude of 20mm. The maximum force which acts upon it is 0.064N. Calculate (a) its maximum velocity (b) its period of oscillation.
4. (a) A body moving with simple harmonic motion in a straight line has a velocity  $v$  and acceleration,  $a$ , when the instantaneous displacement,  $x$  in cm, from its maximum position is given by:  $x = 2.5 \sin 0.4\pi t$ . Determine the magnitude of maximum; (i) velocity (ii) acceleration
- (b) A mass  $m$  attached to a light spiral spring is caused to perform simple harmonic motion of frequency  $f = \frac{2}{2\pi} \sqrt{\frac{k}{m}}$ , where  $k$  is force constant of the spring (i) If  $m=0.30\text{kg}$ ,  $k=30\text{Nm}^{-1}$  and the maximum displacement of the mass from the equilibrium position is 0.015m, calculate the maximum force acting on the system (ii) calculate the maximum kinetic energy of the system (iii) calculate the maximum tension in the spring during the motion [ $g=10\text{ms}^{-2}$ ,  $\pi=3.142$ ]

## WEEKS 10 & 11

### MACHINES

#### CONTENT

- Definition
- Terminologies used in machines
- Types and examples

Machines make our work simpler. It is a force producing device by which a large force called load can be overcome by a small applied force called effort

#### Terminologies Used In Machines

1. FORCE RATIO (MECHANICAL ADVANTAGE )
2. VELOCITY RATIO
3. EFFICIENCY

#### MECHANICAL ADVANTAGE

We define effort as the force applied to a machine and load as the resistance overcome by the machine. The ability of a machine to overcome a large load through a small effort is known as its mechanical advantage .It is given by

$$M.A = \text{Load} / \text{Effort}$$

The mechanical advantage of a machine is influenced by friction in parts

#### VELOCITY RATIO (V.R)

The velocity ratio is the ratio of distance moved by the effort and load in the same interval

$$V.R = \frac{\text{Distance moved by effort}}{\text{Distance moved by the load}}$$

The velocity ratio depends on the geometry of the machine

#### EFFICIENCY (E)

The efficiency of a machine is defined as

$$E_f = \frac{\text{Useful work done by the machine}}{\text{Work put into the machine}} \times 100$$

Work put into the machine

Work = force x distance

$$E_f = \frac{\text{load x distance moved by load}}{\text{Effort x distance moved by effort}} \times 100$$

Effort x distance moved by effort

Then V.R = M.A

## TYPES OF MACHINES

### 1. LEVER

This is the simplest form of machine. It consists of a rigid rod pivoted about a point called the fulcrum F with a small effort applied at one end of the lever to overcome a large load L at the other end. There are various types of lever depending on the relative positions of the load, effort and fulcrum.

Taking moment about F

$E \times a = L \times b$  which is given

$$L = \frac{E \times a}{b} = \text{M.A}$$

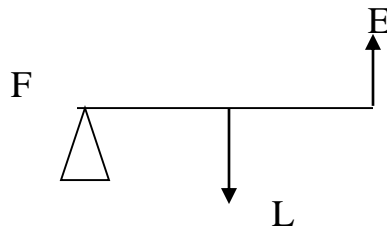
$$E = \frac{L \times b}{a}$$

$$a/b = \text{V.R}$$

Examples of first class lever are the crowbar, pair of scissors or pincers, claw hammer, see-saw, pliers etc

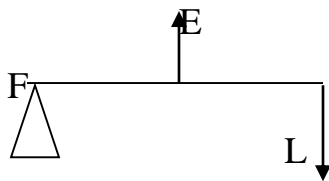


In second order lever , the load is between the fulcrum and effort



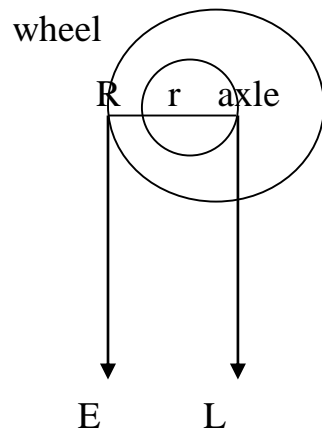
Examples are wheel barrow, nut cracker tarp door , an oar etc .

In the third order lever, the effort is between the fulcrum and the load . Human fore arm , laboratory tong etc.



## 2. WHEEL AND AXLE

It consists of a large wheel to which a rope or string is attached and an axle or small wheel with the rope or string wound round it in opposite direction . The load to be lifted is hung at the free end of the rope on the axle while the effort is applied at the end of the rope on the wheel . For each complete rotation the load and the effort move through distance equal to the circumference of the wheel and axle respectively.



$$V.R = R/r$$

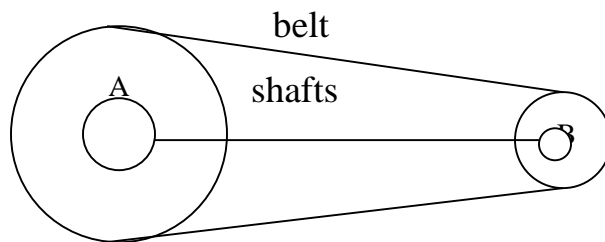
The principle of wheel and axle is used in brace screw driver but spanner windless and gear-boxes

### 3. GEAR WHEELS

In gear boxes , there are toothed wheels of different diameter interlocked to give turning force at low speed depending on which gear is the driver and which is the driven

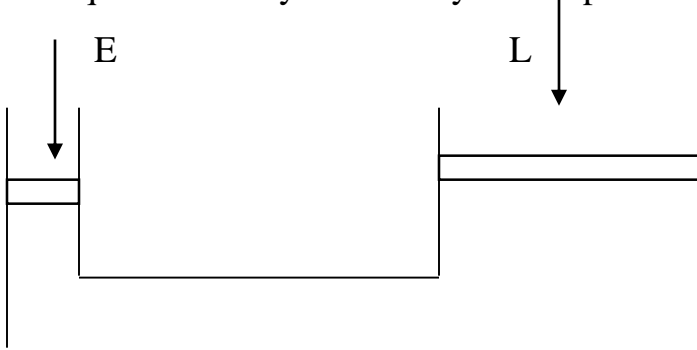
$$V.R = \frac{\text{No of teeth on driven wheel (A)}}{\text{No of teeth on driving wheel (B)}}$$

No of teeth on driving wheel (B)



### 4. THE HYDRAULIC PRESS

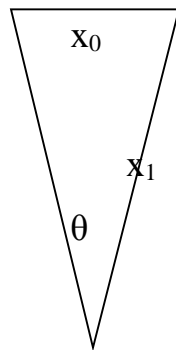
The machine is widely used for compressing waste paper and cotton into compact bales forging different alloys into desirable shape etc .It s work is based on Pascal's principle which states that pressure is transmitted equally in fluid Oil is the liquid normally used in hydraulic press



$$V.R = R^2/r^2$$

## 5. THE WEDGE

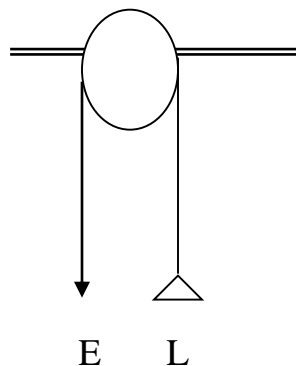
The wedge is a combination of two inclined planes. It is used to separate bodies which are held together by large force. Examples of wedge type of machines are axes chisels knives etc.



$$M.A = \frac{X_1}{X_0} = \frac{\text{Slant height of wedge}}{\text{Thickness of wedge}}$$

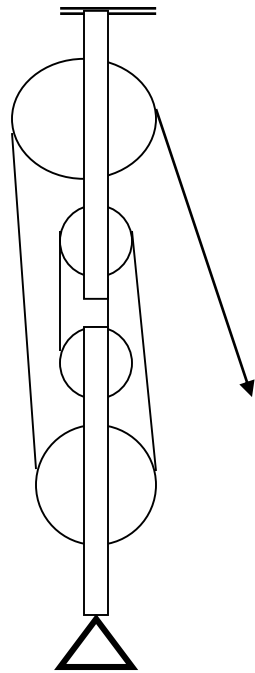
## 6. PULLEY

A simple pulley is a fixed wheel hung on a suitable support with a rope passing round its groove.

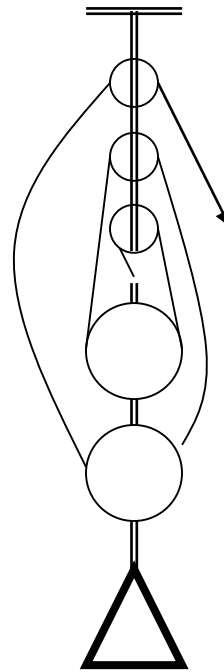


**BLOCK AND TACKLE (PULLEY)**

This is the more practical system of pulleys in which one or more pulley are mounted on the same axle with one continuous rope passing all-round the pulleys



$$V.R = 4$$



$$V.R = 5$$

### EFFECTS OF FRICTION ON MACHINE

Work is always wasted in machines to overcome the frictional forces present between the moving parts and also to lift to part of the machine. The greater the friction, the greater the effort required and the smaller the M.A. M.A depend on friction but depend on the geometry of moving parts.

The efficiency of nearly all the machines varies with the load and the load and effort are related by :  $E = al + b$  (  $a$  and  $b$  are constant ). This is called linear law for a machine .It follows that  $E$  is proportional to  $L$  .The value to give us the

effort required to operate the machine moving part only if no load is present while A gives us the measure of the friction present

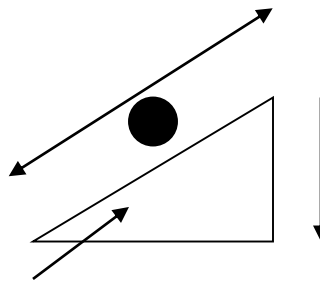
$$= \frac{M.A \times 100}{V.R}$$

In practical machines the efficiency is usually less than 100% because of friction in the moving parts of the machine.

(1) INCLINED PLANE: This is in form of a sloping plank commonly used to raise heavy load such as barrels of oil with little applied effort than by lifting it vertically .

x

h



$$V.R = \frac{\text{Distance moved by effort}}{\text{Distance moved by load}}$$

$$= x / h ; V.R = 1 / \sin \theta$$

## THE SCREW

Geometrically speaking the screw is an inclined plane wrapped round a cylinder to form a thread. The distance between successive threads on a screw is called its pitch. For one complete revolution of screw through an effort , the load moves a distance equal to its pitch e.g. screw jack nut and bolt

In a screw jack where length of the operating handle is a, the effort moved a distance equal to the pitch P.

$$\text{Thus } V.R = \frac{2\pi a}{P}$$

$$= \frac{2\pi r}{P}$$

If frictional forces are negligible

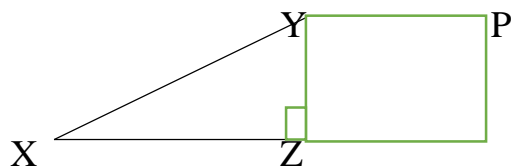
### CLASSWORK

1. (a) What is a machine? (b) Explain why a machine can never be 100% efficient.
2. Define the following terms as applicable to machine (a) velocity ratio (ii) mechanical advantage (iii) efficiency
3. A pulley with velocity ratio of 5 is used to lift a load of 400N through a vertical height of 8m by exerting an effort of 100N. Calculate the: (a) work done by the effort (b) efficiency of the pulley system

### ASSIGNMENT

#### SECTION A

1. The statement that the mechanical advantage of machine is 3 means that the (a) efficiency is  $33\frac{1}{3}$  (b) effort is three times as large as the load (c) mechanical advantage is three times as large as velocity ratio (d) ratio of effort to load is 1:3
2. In the diagram below XY represents a plank used to lift a load from a point X on the ground onto a horizontal platform YP.



What is the velocity ratio of the plank? (a)  $\frac{XY}{ZY}$  (b)  $\frac{XY}{XZ}$  (c)  $\frac{XZ}{XY}$  (d)  $\frac{ZY}{XY}$

3. A machine with a velocity ratio of 30 moves a load of 3000N when an effort of 200N is applied. The efficiency of the machine is (a) 30% (b) 50% (c) 60% (d) 75%

4. The efficiency of a wheel and axle system is 80% and the ratio of radius of wheel to radius of the axle is 4 : 1, In order to lift a mass of 20kg, the effort required is (a) 60N (b) 62.5N (c) 32.5N (d) 250
5. The velocity ratio of an inclined plane whose angle of inclination is  $\theta$  is (a)  $\sin \theta$  (b)  $\cos \theta$  (c)  $\tan \theta$  (d)  $1/\sin \theta$
6. Which of the following is not an example of levers of the first order? (a) crow bar (b) Nutcracker (c) scissors (d) pliers
7. A body of mass 7.5kg is to be pulled up along a plane which is inclined at  $30^\circ$  to the horizontal. If the efficiency of the plane is 75%, what is the minimum force required to pull the body up the plane? [ $g=10\text{ms}^{-2}$ ] (a) 5.0N (b) 20.0N (c) 50.0N (d) 200.0N
8. Calculate the velocity ratio of a screw jack of pitch 0.3cm if the length of the tommy bar is 21cm (a)  $\frac{1}{140}\pi$  (b)  $14\pi$  (c)  $70\pi$  (d)  $140\pi$
9. A machine with a velocity ratio of 5 is used to raise a load with an effort of 500N. If the machine is 80% efficient, determine the magnitude of the load (a) 2500N (b) 2000N (c) 1200N (d) 625N
10. A block and tackle system of pulley has 6 pulleys. If the efficiency of the machine is 60%, determine its mechanical advantage (a) 12.0 (b) 10.0 (c) 3.6 (d) 1.8

## SECTION B

1. Show that efficiency  $E$ , the force ratio (MA) and the velocity ratio (VR) of a machine are related by the equation  $E = \frac{MA}{VR} \times 100\%$
2. (a) Draw a diagram of a system of pulleys with a velocity ratio of 5 (b) A man pulls up a box of mass 70kg using an inclined plane of effective length 5m onto a platform 2.5m high at uniform speed. If the frictional force between the box and plane is 100N, draw the diagram of forces acting on

the box when in motion and calculate the; (i) minimum effort applied in pulling up the box (ii) velocity ratio of the plane (iii) mechanical advantage of the plane (iv) efficiency of the plane (v) energy lost in the system (vi) work output of the man (vii) total power developed by the man given that the time taken to raise the box onto the platform is 50seconds [ $g=10\text{ms}^{-2}$ ]

3. A screw jack, 25% efficient and having a screw of pitch 0.4cm is used to raise a load through a certain height. If in the process the handle turns through a circle of radius 40cm, calculate the (a) velocity ratio of the machine (b) the mechanical advantage of the machine (c) effort required to raise a load of 100N with the machine [ $\pi=3.14$ ]



**WEEK 12**

*Revision*

**WEEK 13**

*Examination*